Final Exam

MAS501 Analysis for Engineers, Spring 2011

♠ 공지사항 ♠

- 휴대전화는 진동으로 맞춰 주시고 시간을 확인하는 용도로만 사용해 주십시오.
- 학번과 이름을 기입하신 후 답안지는 양면으로 작성해 주십시오.
- 고득점의 비결은 간결하면서도 명료한 답안을 작성하는 것입니다.
- 채점 결과는 5/31일(화)에 강의 홈페이지를 통해 공개됩니다. 5/31(화)부터 6/3(금) 오후 2 시에서 3시 사이에 강사 오피스에 방문하셔서 답안지를 돌려 받거나 클레임을 해주십시오.

1. (45pts) Determine whether the following statements are true or false. (You don't need to prove or disprove.) If you are correct, you gain three points. If you are wrong, you *lose* three points. If you don't write the answer, nothing happens.

   (a) If $|f|$ is Riemann integrable, then f is Riemann integrable.

   (b) If f is increasing and α is continuous on $[a,b]$, then the Riemann-Stieltjes integral $\int_a^b f\,d\alpha$ exists.

   (c) There is a continuously differentiable function f on $(0,1)$ such that f' is bounded but f is *not* bounded.

   (d) Let $f(x) = x^{3/2}\sin(1/x)$ when $x \neq 0$ and $f(0) = 0$. Then f is of bounded variation on $[0,1]$.

   (e) If f is a continuously differentiable function on $[a,b]$, $f(a) = f(b) = 0$ and $\int_a^b f^2(x)\,dx = 1$, then $\int_a^b xf(x)f'(x)\,dx < 0$.

   (f) It holds that $\int_{-1}^2 x\sqrt{1+x^2}\,dx > 1/3$.

   (g) Let f be a continuous function. Then $\lim_{h\to 0}\int_{a-h}^{a+h} f(x)\,dx = f(a)$.

   (h) Let $\{f_n\}$ be a sequence of continuous functions. If $f_n \to f$ uniformly on $\mathbf{R}$, then f is continuous on $\mathbf{R}$.

   (i) $f(x) = \sum_{n=1}^\infty \sin(nx)/n^2$ is a continuous function on $\mathbf{R}$.

   (j) A bounded continuous function on $\mathbf{R}$ is differentiable at some point in $\mathbf{R}$.

   (k) Let $\{f_n\}$ be an equicontinuous sequence of functions on $\mathbf{R}$. If $f_n \to f$ pointwise on $\mathbf{R}$, then f is uniformly continuous on $\mathbf{R}$.

   (l) There is a countable set which is not measurable.

   (m) Let f be a real-valued function on $\mathbf{R}$. If $\{x : f(x) = \alpha\}$ is measurable for every real number α, then f is measurable.

   (n) If $\{f_n\}$ is a sequence of continuous functions on $\mathbf{R}$, then $\limsup f_n$ is measurable.

   (o) If $\{f_n\}$ is a sequence of nonnegative measurable functions and $f_n \to f$ a.e. on a measurable set E, then $\int_E f\,dm \le \liminf \int_E f_n\,dm$.

2. (10pts) Find the *smallest* real number a satisfying

$$x - \ln(1+x) \le ax^2 \quad \text{for every } x \ge 0.$$

You should justify your answer.

3. (10pts) Prove that the improper integral

$$\int_0^\infty \big(1+\sin(nx)\big)\Big(1+\frac{x}{n}\Big)^n e^{-2x}\,dx$$

exists for every natural number n. DO NOT use the Lebesgue integral.

4. Assume $f_n \to f$ uniformly and $g_n \to g$ uniformly on $\mathbf{R}$.

   (a) (5pts) Prove or disprove: $f_n + g_n \to f + g$ uniformly on $\mathbf{R}$.

   (b) (5pts) Prove or disprove: $f_n g_n \to fg$ uniformly on $\mathbf{R}$.

5. (10pts) Prove or disprove: if $\{E_n\}$ is an *increasing* sequence of measurable sets, that is, $E_n \subset E_{n+1}$ for each n, then

$$m\Big(\bigcup_{n=1}^\infty E_n\Big) = \lim_{n\to\infty} m(E_n).$$

6. (10pts) Compute

$$\lim_{n\to\infty}\int_0^n n(e^{-x-1/n} - e^{-x})\,dx.$$

You should justify your answer.

7. (5pts) 강의에 관한 설문 조사입니다. 해당하는 개념이나 정리를 적어주세요.

   (a) 가장 흥미로웠던 것.

   (b) 본인에게 가장 유용할 것 같은 것.

   (c) 이해하는데 가장 힘들었던 것.